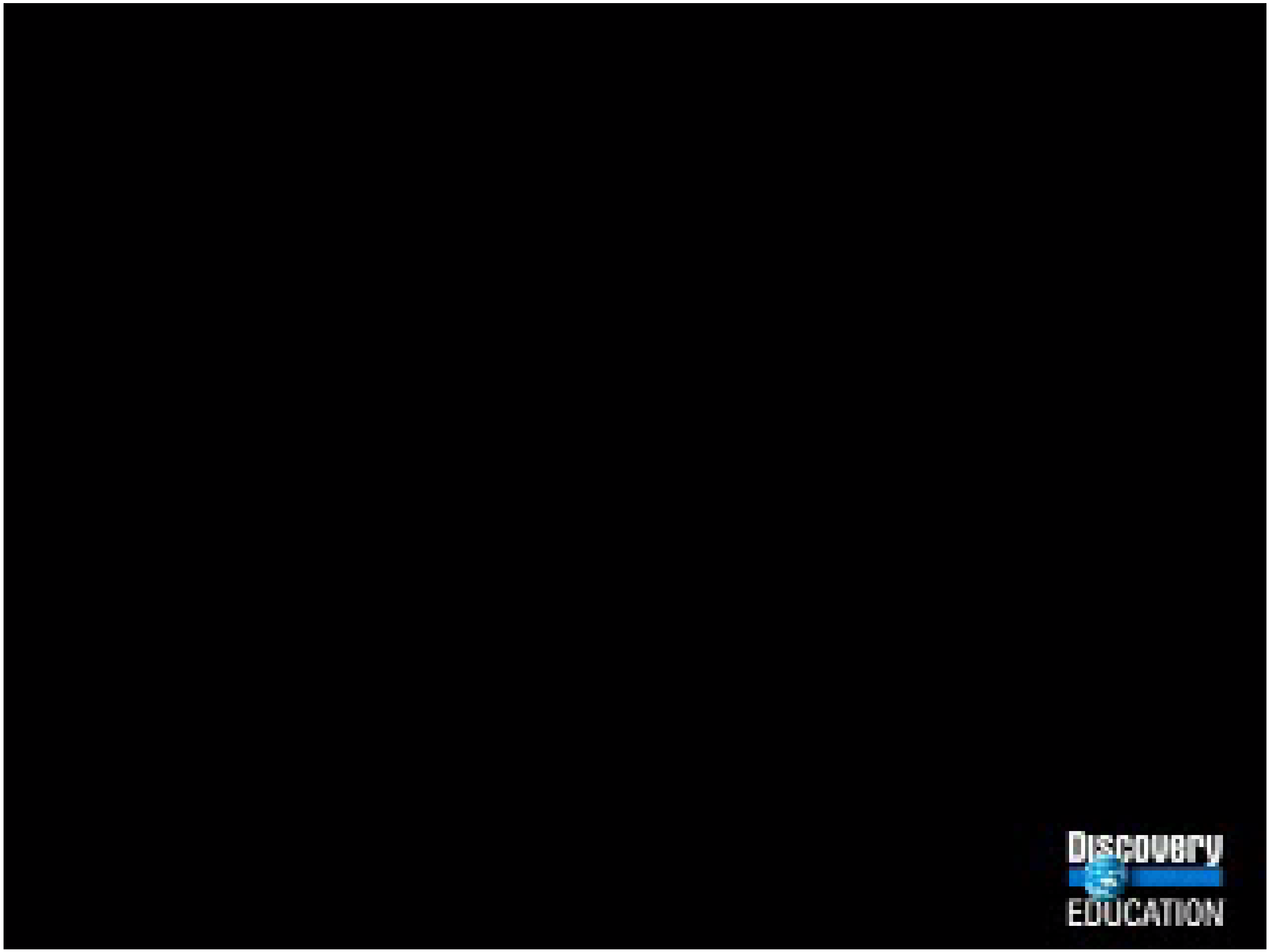
Lesson 2.3

MODELING REAL WORLD DATA WITH MATRICES

Matrix Arithmetic



EXAMPLE 1 **Find the values of *x* and *y* for which the matrix equation**

$$\begin{bmatrix} y \\ x \end{bmatrix} = \begin{bmatrix} 3x + 16 \\ 3y \end{bmatrix}$$ **is true.**

Since the corresponding elements are equal, we can express the equality of the matrices as two equations.

$y = 3x + 16$
$x = 3y$

$y = 3x + 16$
$y = 3(3y) + 16$
$y = -2$

$x = 3(-2)$
$x = -6$

The matrices are equal if $x = -6$ and $y = -2$.

Key Concept — Adding and Subtracting Matrices

For Your FOLDABLE

Words To add or subtract two matrices with the same dimensions, add or subtract their corresponding elements.

Symbols

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} + \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{A+B}{\begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}}$$

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} - \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{A-B}{\begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}}$$

Example

$$\begin{bmatrix} 3 & -5 \\ 1 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -9 & 10 \end{bmatrix} = \begin{bmatrix} 3+2 & -5+0 \\ 1+(-9) & 7+10 \end{bmatrix}$$

EXAMPLE 2

Find $A + B$ if $A = \begin{bmatrix} 4 & -2 & 6 \\ 1 & 3 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 5 \\ -4 & 1 & 7 \end{bmatrix}$.

$$A + B = \begin{bmatrix} 4+(-1) & -2+2 & 6+5 \\ 1+(-4) & 3+1 & -3+7 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 11 \\ -3 & 4 & 4 \end{bmatrix}$$

EXAMPLE 2

Find $C - D$ if $C = \begin{bmatrix} 5 & 2 \\ 8 & 1 \\ -4 & 3 \\ 2 & -1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 5 \\ -3 & 4 \\ 6 & -8 \\ 3 & 5 \end{bmatrix}$.

C* – *D* = *C* + -*D

$$\begin{bmatrix} 5 & 2 \\ 8 & 1 \\ -4 & 3 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} -2 & -5 \\ 3 & -4 \\ -6 & 8 \\ -3 & -5 \end{bmatrix} = \begin{bmatrix} 5+(-2) & 2+(-5) \\ 8+3 & 1+(-4) \\ -4+(-6) & 3+8 \\ 2+(-3) & -1+(-5) \end{bmatrix} \text{ or } \begin{bmatrix} 3 & -3 \\ 11 & -3 \\ -10 & 11 \\ -1 & -6 \end{bmatrix}$$

Key Concept — Multiplying by a Scalar

For Your FOLDABLE

Words To multiply a matrix by a scalar k, multiply each element by k.

Symbols

$$k \cdot A = kA$$

$$k\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$$

Example

$$-3\begin{bmatrix} 4 & 1 \\ 7 & -2 \end{bmatrix} = \begin{bmatrix} -3(4) & -3(1) \\ -3(7) & -3(-2) \end{bmatrix}$$

EXAMPLE 3 **If** $A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 5 & 0 \\ 3 & 6 & 2 \end{bmatrix}$**, find 2**$A$**.**

$$2\begin{bmatrix} 1 & 3 & 4 \\ -2 & 5 & 0 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2(1) & 2(3) & 2(4) \\ 2(-2) & 2(5) & 2(0) \\ 2(3) & 2(6) & 2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 6 & 8 \\ -4 & 10 & 0 \\ 6 & 12 & 4 \end{bmatrix}$$

Key Concept — Multiplying Matrices

For Your FOLDABLE

Words The element in the *m*th row and *r*th column of matrix *AB* is the sum of the products of the corresponding elements in row *m* of matrix *A* and column *r* of matrix *B*.

Symbols

$$\underset{A}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} \cdot \underset{B}{\begin{bmatrix} e & f \\ g & h \end{bmatrix}} = \underset{AB}{\begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}}$$

Multiplying Matrices

EXAMPLE 4

Multiply Square Matrices

Step 1 Multiply the numbers in the first row of *R* by the numbers in the first column of *S*, add the products, and put the result in the first row, first column of *RS*.

$$\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \bullet \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3(-2)+2(1) & \\ & \end{bmatrix}$$

EXAMPLE 4 **Multiply Square Matrices**

Step 2 Multiply the numbers in the first row of *R* by the numbers in the second column of *S*, add the products, and put the result in the first row, second column of *RS*.

$$\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \bullet \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3(-2)+2(1) & 3(1)+2(-1) \\ & \end{bmatrix}$$

EXAMPLE 4

Multiply Square Matrices

Step 3 Multiply the numbers in the second row of *R* by the numbers in the first column of *S*, add the products, and put the result in the second row, first column of *RS*.

$$\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \bullet \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3(-2)+2(1) & 3(1)+2(-1) \\ -1(-2)+0(1) & \end{bmatrix}$$

EXAMPLE 4

Multiply Square Matrices

Step 4 Multiply the numbers in the second row of *R* by the numbers in the second column of *S*, add the products, and put the result in the second row, second column of *RS*.

$$\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} \bullet \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 3(-2)+2(1) & 3(1)+2(-1) \\ -1(-2)+0(1) & -1(1)+0(-1) \end{bmatrix}$$

EXAMPLE 4

Multiply Square Matrices

Step 5 Simplify the product matrix.

$$\begin{bmatrix} 3(-2)+2(1) & 3(1)+2(-1) \\ -1(-2)+0(1) & -1(1)+0(-1) \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}$$

Answer: So, $RS = \begin{bmatrix} -4 & 1 \\ 2 & -1 \end{bmatrix}$.

EXAMPLE 4

Use matrices $A = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & -1 \end{bmatrix}$, **and** $C = \begin{bmatrix} -3 & 4 & 2 \\ 1 & 5 & 0 \end{bmatrix}$ **to find each product.**

a. ***AB***

$$AB = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2(3)+4(4) & 2(1)+4(0) & 2(-2)+4(-1) \\ 0(3)+1(4) & 0(1)+1(0) & 0(-2)+1(-1) \end{bmatrix} \text{ or } \begin{bmatrix} 22 & 2 & -8 \\ 4 & 0 & -1 \end{bmatrix}$$

b. ***BC***

B is a 2×3 matrix and C is a 2×3 matrix. Since B does not have the same number of columns as C has rows, the product BC does not exist. BC is undefined.

Example 5.

SHOPPING At a certain clothing store, each pair of jeans (J) is priced at $15, each t-shirt (T) is priced at $10, and each sweater (S) is priced at $20. The chart lists the number of each of these items purchased by five shoppers. Use matrix multiplication to find the total amount spent by each shopper.

Shopper	Jeans	T-Shirts	Sweaters
Sarah	1	3	1
Dave	2	2	0
Jessica	3	1	2
Drew	0	4	1
Emily	1	2	2

Example 5.

Write the purchase information as a 5 × 3 matrix and write the prices as a 3 × 1 matrix. Then multiply the matrices.

$$\begin{array}{l} \\ \text{Sarah} \\ \text{Dave} \\ \text{Jessica} \\ \text{Drew} \\ \text{Emily} \end{array} \begin{array}{c} \begin{array}{ccc} J & T & S \end{array} \\ \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 0 \\ 3 & 1 & 2 \\ 0 & 4 & 1 \\ 1 & 2 & 2 \end{bmatrix} \end{array} \cdot \begin{array}{c} \\ J \\ T \\ S \end{array} \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 15 \\ 10 \\ 20 \end{bmatrix} \end{array} =$$

$$\begin{array}{l} \\ \text{Sarah} \\ \text{Dave} \\ \text{Jessica} \\ \text{Drew} \\ \text{Emily} \end{array} \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 1(15) + 3(10) + 1(20) \\ 2(15) + 2(10) + 0(20) \\ 3(15) + 1(10) + 2(20) \\ 0(15) + 4(10) + 1(20) \\ 1(15) + 2(10) + 2(20) \end{bmatrix} \end{array} = \begin{array}{l} \\ \text{Sarah} \\ \text{Dave} \\ \text{Jessica} \\ \text{Drew} \\ \text{Emily} \end{array} \begin{array}{c} \text{Cost} \\ \begin{bmatrix} 65 \\ 50 \\ 95 \\ 60 \\ 75 \end{bmatrix} \end{array}$$