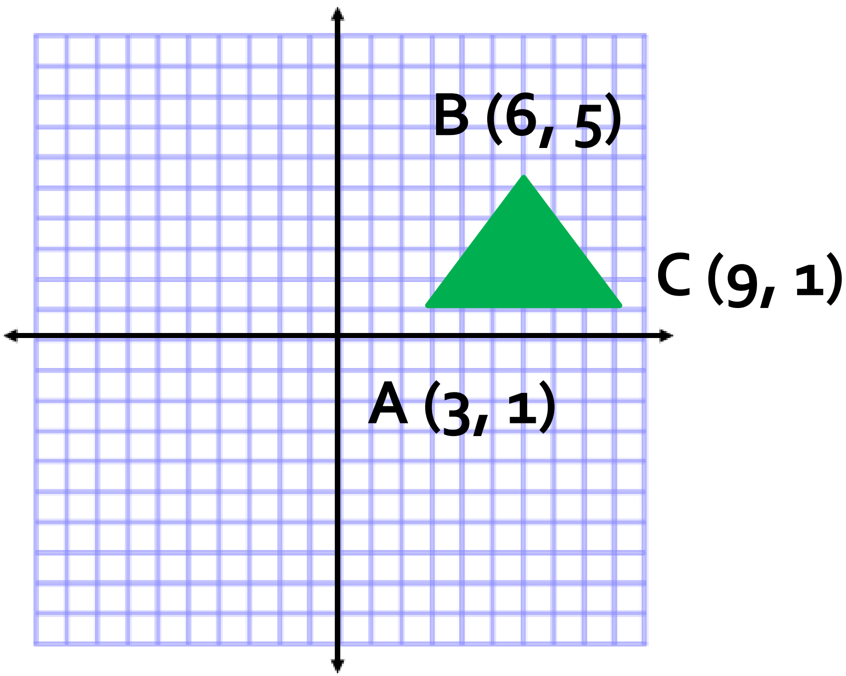
Lesson 2.4

Modeling Motions with Matrices.

A **2 x n** matrix can be used to express the vertices of an **n – gon** with the first row of elements representing the x-coordinates and the y-coordinates of the vertices the second row.



$$\begin{array}{r} \\ \text{x-coordinates} \\ \text{y-coordinates} \end{array} \begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 3 & 6 & 9 \\ 1 & 5 & 1 \end{bmatrix} \end{array}$$

Translation

A translation "slides" an object a fixed distance in a given direction. The original object and its translation have the same shape and size, and they face in the same direction.
The word "translate" in Latin means "carried across".

An original matrix is called a pre-image. A matrix after translation is called an image.

Example 1 Translation

Determine the coordinates of the vertices of the image of quadrilateral *ABCD* with *A*(–5, –1), *B*(–2, –1), *C*(–1, –4), and *D*(–3, –5), if it is translated 3 units to the right and 4 units up. Then graph *ABCD* and its image *A′B′C′D′*.

Write the vertex matrix for quadrilateral *ABCD*.

$$\begin{bmatrix} -5 & -2 & -1 & -3 \\ -1 & -1 & -4 & -5 \end{bmatrix}$$

Example 1 Translation

$$\underset{\text{of } ABCD}{\text{Vertex Matrix}} \quad \underset{\text{Matrix}}{\text{Translation}} \quad \underset{\text{of } A'B'C'D'}{\text{Vertex Matrix}}$$

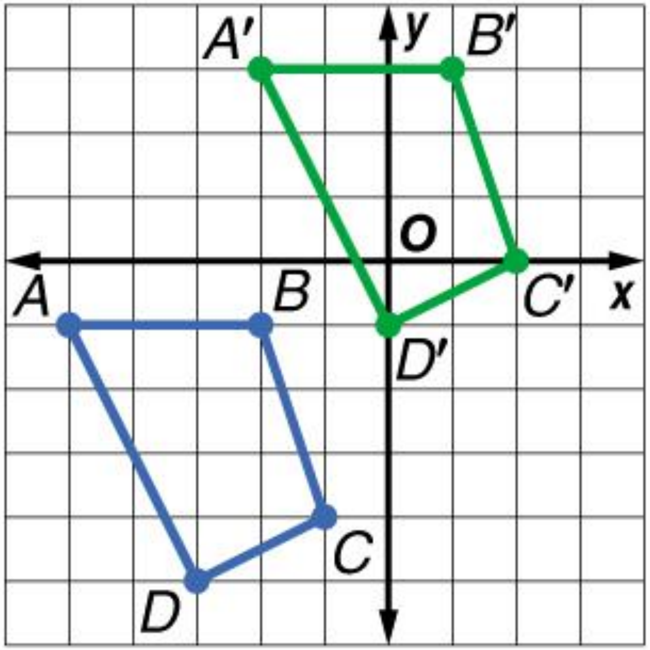
$$\begin{bmatrix} -5 & -2 & -1 & -3 \\ -1 & -1 & -4 & -5 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 2 & 0 \\ 3 & 3 & 0 & -1 \end{bmatrix}$$

The coordinates of $A'B'C'D'$ are $A'(–2, 3)$, $B'(1, 3)$, $C'(2, 0)$, $D'(0, –1)$.

Graph the preimage and the image. The two graphs have the same size, shape and orientation.

Example 1 Translation

Answer: *A′*(–2, 3), *B′*(1, 3), *C′*(2, 0), *D′*(0, –1)



Dilations

- A dilation is a transformation that produces an image that is the **same shape** as the original, but is **a different size.**
- A dilation "**shrinks**" (a scale factor is **less than 1**) or "**stretches"** (a scale factor is **greater than 1**) a figure.

Example 2 **Dilation**

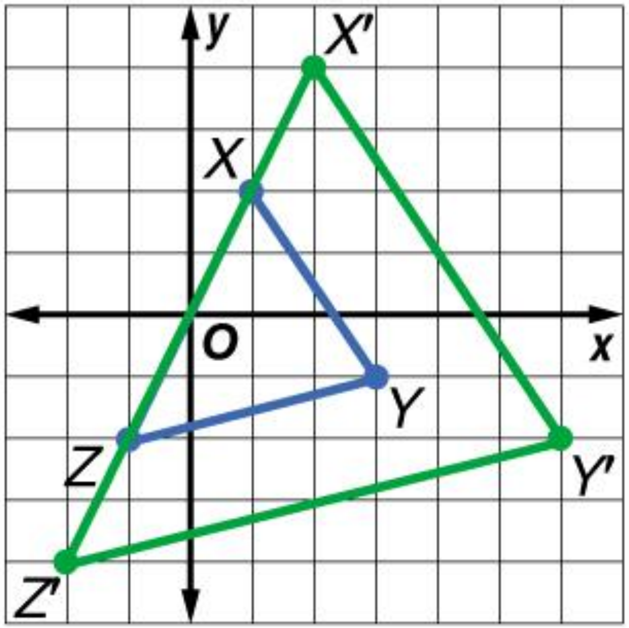
ΔXYZ has vertices $X(1, 2)$, $Y(3, -1)$, and $Z(-1, -2)$. Dilate ΔXYZ so that its perimeter is twice the original perimeter. Find the coordinates of the vertices of $\Delta X'Y'Z'$. Then graph ΔXYZ and $\Delta X'Y'Z'$

Multiply the vertex matrix by the scale factor of 2.

$$2\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 6 & -2 \\ 4 & -2 & -4 \end{bmatrix}$$

Example 2 Dilation

The coordinates of the vertices of $\Delta X'Y'Z'$ are $X'(2, 4)$, $Y'(6, -2)$, and $Z'(-2, -4)$.



Graph ΔXYZ and $\Delta X'Y'Z'$. $\Delta X'Y'Z'$ has sides that are twice the length of those of ΔXYZ.

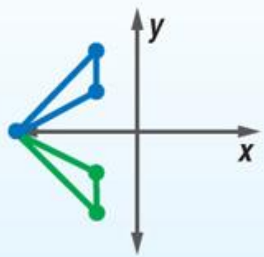
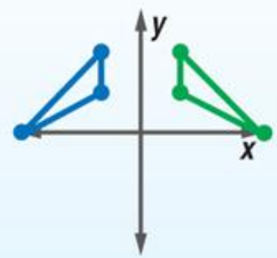
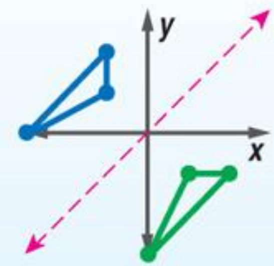
Answer: The coordinates of the vertices of $\Delta X'Y'Z'$ are $X'(2, 4)$, $Y'(6, -2)$, and $Z'(-2, -4)$. The preimage and image are similar. Both figures have the same shape.

Key Concept

Reflection Matrices

For Your FOLDABLE

To reflect in the given line, multiply the vertex matrix by the given matrix.

Line of Reflection	x-axis	y-axis	line $y = x$
Multiply on the left by:	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Models			

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Example 3 Reflection

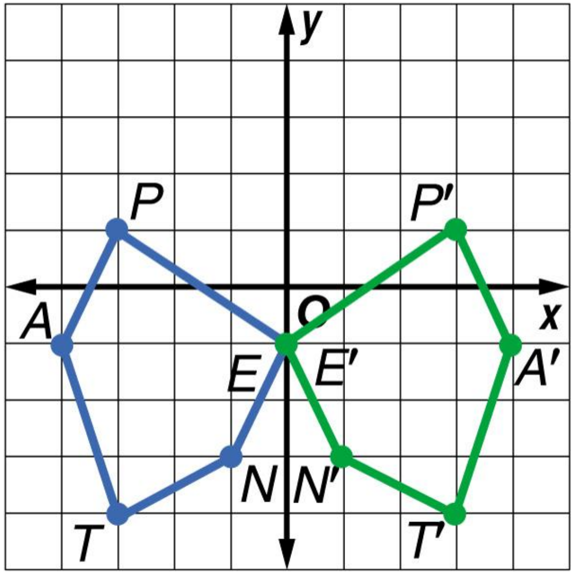
Determine the coordinates of the vertices of the image of pentagon *PENTA* with *P*(–3, 1), *E*(0, –1), *N*(–1, –3), *T*(–3, –4), and *A*(–4, –1) after a reflection across the *y*-axis. Then graph the preimage and image.

Write the ordered pairs as a vertex matrix. Then multiply the vertex matrix by the reflection matrix for the *y*-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} -3 & 0 & -1 & -3 & -4 \\ 1 & -1 & -3 & -4 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 & 3 & 4 \\ 1 & 1 & -3 & -4 & -1 \end{bmatrix}$$

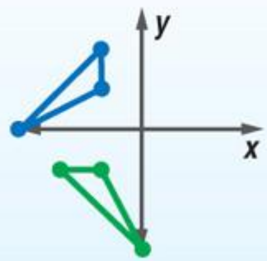
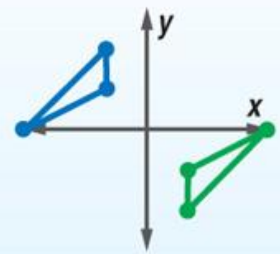
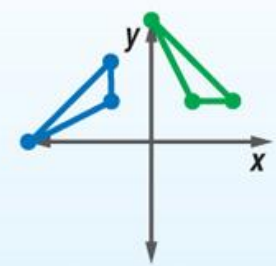
Example 3 Reflection

Answer: The coordinates of the vertices of *P′E′N′T′A′* are *P′*(3, 1), *E′*(0, –1), *N′*(1, –3), *T′*(3, –4), and *A′*(4, –1). Notice that the preimage and image are congruent. Both figures have the same size and shape.



Key Concept **Rotation Matrices** For Your FOLDABLE

To rotate counterclockwise about the origin, multiply the vertex matrix by the given matrix.

Angle of Rotation	90°	180°	270°
Multiply on the left by:	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
Models			

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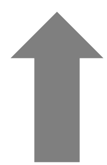
The Sky Is The Limit



Example 4 **Rotation**

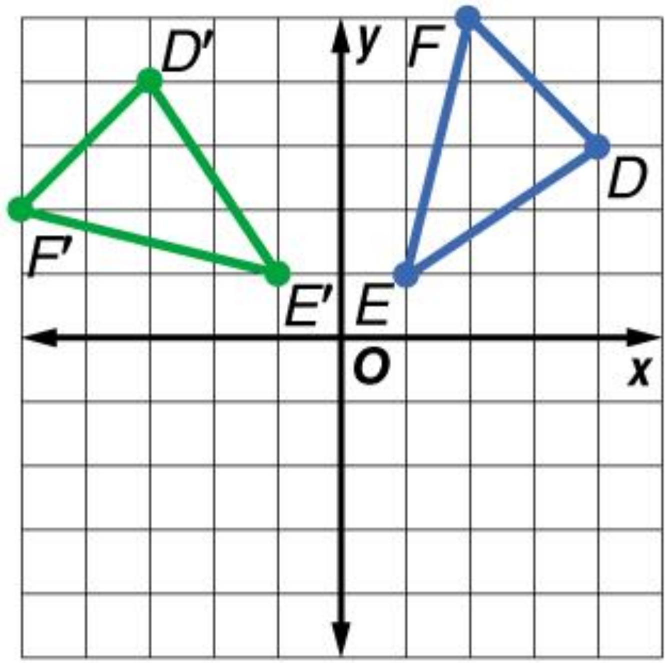
Find the coordinates of the vertices of the image of Δ*DEF* with *D*(4, 3), *E*(1, 1), and *F*(2, 5) after it is rotated 90° counterclockwise about the origin.

Write the ordered pairs in a vertex matrix. Then multiply the vertex matrix by the rotation matrix.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 4 & 1 & 2 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} -3 & -1 & -5 \\ 4 & 1 & 2 \end{bmatrix}$$

Example 4 Rotation

Answer: The coordinates of the vertices of triangle *D′E′F′* are *D′*(–3, 4), *E′*(–1, 1), and *F′*(–5, 2). The image is congruent to the preimage.



Symmetry and Transformations.